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*Strategic Electoral Choice in Multi-Member Districts: Approval Voting in Practice?**

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The chief aim of this paper is to elucidate the theory of electoral choice in an important but previously neglected electoral context: multi-member districts. Taking the simplest case of a three-candidate election in a double-member district, the paper develops a model of electoral choice as a standard decision under risk. It is found that this simplest case is essentially equivalent to approval voting with three candidates; thus, voting in double elections with three candidates is "sincere" in the same sense that approval voting is, and a unique sincere strategy exists when preferences are dichotomous. Nonetheless, it is shown that electoral choice in double (and approval) elections is inherently strategic in the sense that voters' beliefs about how others will vote affect their decisions (when preferences are not dichotomous). This point is important because it has been claimed that sincerity of choice is an important property, guaranteeing that choices reflect preferences "directly." This paper shows that there are fairly severe restrictions on the sense in which this is true, due to the strategic nature of choice. In the later sections of the paper, an examination of actual voting behavior in double-member districts is made to see if theoretically predicted strategic behavior is evident in practice. The evidence is positive.

Although unknown at the federal level, elections at which two or more legislators are simultaneously chosen from the same geographic district are frequently used in states and localities. As of 1970, over 60 percent of the cities and 30 percent of the counties in the United States elected their legislative bodies at-large rather than by single-member districts (Jewell, 1970, p. 52). In the lower houses of the 49 states having bicameral legislatures, over 35 percent of the 5,501 members in 1980 were returned from multi-member districts. Nine years earlier, over 50 percent of state representatives were elected in multiple constituencies, and in 1962, just before the landmark *Baker v. Carr* decision, the comparable figure was 46 percent (Council of State Governments, 1971, 1980; David and Eisenberg, 1962). Multi-member districts (MMDs) were in even more frequent use earlier; Klain notes that "for nearly a century after the Declaration of Independence the American states elected by far the greater part of their law-makers in multiple constituencies" (Klain, 1955, p. 1113). This earlier preponderance of MMDs was largely an inheritance from England, where, down until the massive redistricting of 1885, the modal category of district was double-member.

The political effects of multiple as opposed to single elections have long been argued by political scientists and other observers. The single-member district (SMD) has often been criticized as promoting a parochial view on the part of legislators, who must cater to what are sometimes narrowly defined local interests. Thus a New York constitutional commission in 1872, arguing that SMDs

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produced excessive amounts of special and local legislation, recommended that the Senate be elected from MMDs (Silva, 1964, p. 507). Lord Bryce and others supportive of the old double-member system in England argued that, on both sides of the Atlantic, a decline in the quality of legislators was attributable to the adoption of the SMD: "The area of choice being smaller, inferior men are chosen" (Bryce, 1889, pp. 463–64, cited in Klain, 1955, p. 1118). Recently, multiple (especially at-large) elections in the United States have been attacked in the courts as methods of denying racial and/or political minorities proportionate representation:

> The "winner-take-all" character of the typical election scheme creates the possibility that a specific majority will elect all the representatives from a multimember district whereas the outvoted minority might have been able to elect some representatives if the multimember district had been broken down into several single-member districts. (Tribe, 1978, p. 750)

Surprisingly, in the relatively long history of discussion of the effects of the MMD, there has been very little scholarly attention given to voting behavior, either from an empirical or a theoretical perspective. Yet, some knowledge of voting behavior would seem to be a prerequisite to any complete analysis of the other political effects produced by MMDs.

Consider, for example, the fate of political (i.e., partisan) minorities in multiple elections. Hamilton (1967) finds that only 4 of 45 MMD elections held in Indiana between 1954 and 1962, inclusive, resulted in the return of both Republican(s) and Democrat(s). In Michigan, all 85 multiple elections in the same period were swept by one party. These results would indicate, assuming that at least some of the multi-member districts then used could have been split into SMDs in such a fashion as to produce a party split, that MMDs may deny minority representation. Yet in Ohio, Hamilton finds that "voters appear to vote 'for the man' to [some] extent": 27 of 69 MMD elections—or nearly 40 percent—resulted in split decisions. And in nineteenth century England even higher proportions of multiple elections rendered split decisions. For example, of the 190 double-member elections held at the general elections of 1859 and 1865, 44 percent resulted in the return of one Conservative and one Liberal.

In the remainder of this paper, the focus will be on voting behavior in the simplest possible multi-member situation: three-candidate elections in double-member districts. It should be noted, however, that the basic approach used here can be readily adapted to the study of other small MMD situations—e.g., cumulative voting in Illinois—and could in principle be applied to any MMD election.

Another point which should be made clear at this stage is that the elections dealt with in this paper are true double elections in which voters have the option of casting two votes. A considerable number of multi-member districts operate under the so-called place system, wherein each seat is contested separately (Boyd, 1976; Young, 1965). For example, in a k-member district, there would be k "places," and each candidate would file for exactly one of them, running against

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only those other candidates who had filed for the same place. The place system in effect converts MMDs into a collection of coextensive SMDs. This kind of electoral system is not dealt with here.

In the next section, the method of voting in double-member districts is explained. Then a decision-theoretic model of electoral choice is developed for the three-candidate case. An added bonus in studying this simplest case, beyond tractability, is that three-candidate double-member elections are essentially equivalent to three-candidate elections held under a "negative" approval voting system. Approval voting—a method for conducting multicandidate elections in which a voter may vote for as many candidates as he or she wishes—is being promoted as "the election reform of the 20th century" (Brams, 1980, p. 105). The equivalence of the double-member and approval systems in the three-candidate case means that the empirical evidence regarding voting behavior in double-member districts presented here can give something of a preview of the reform in practice. Further, the theoretical results demonstrated for double-member voting apply to the approval voting case as well.

The theoretical approach taken here is to view electoral choice in double-member districts as a standard decision under risk and to utilize the assumption that voters maximize expected utility in order to generate behavioral predictions. As expected from the asserted equivalence between double and approval voting, it is found that voting in three-candidate double-member elections is "sincere" in the same sense in which approval voting is sincere (Brams and Fishburn, 1978). However, it is also found that there is a species of "strategic" voting endemic to double elections, most closely akin to that identified for approval voting by Niemi (forthcoming [1984]), but related also to that found in the SMD case by McKelvey and Ordeshook (1972) and studied empirically by Cain (1978) and Black (1978). By "strategic" it is meant that voting decisions depend not solely on preferences but also on probability beliefs about how other voters will vote. The empirical part of the paper attempts to assess the practical importance of the strategic behavior which appears quite clearly in the theory. Falsifiable hypotheses are deduced from the model, and these hypotheses are put to the test using a particularly detailed form of electoral documentation (which, unfortunately, is available only for elections in England held before 1948).

The Anglo-American Double-Member District

Before launching into the analysis, the history and workings of the double-member district should be reviewed briefly. The modal type of district in England from the thirteenth through most of the nineteenth century was double-member, with the next most common types being single- and triple-member. Early records from the colonies indicate that representatives to colonial legislatures were returned from the same mix of districts, although there was a tendency also to use larger multi-member districts. As noted above, the predominance of double- and other multi-member districts continued well past the colonial period (Klain, 1955).

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Double-member districts continue to play an important role in a number of state lower houses. In the latest published survey, reporting on the apportionment of legislatures in the 1970s, at least 14 state lower houses still used double-member districts, and three—Arizona, New Jersey, and Washington—relied exclusively on them (Council of State Governments, 1980). Further, a check with election officials of these last three states reveals that Arizona and New Jersey used (and still use) "true" double-member districts, rather than operating under the place system (as did Washington).

In true double-member districts, which like single-member districts cover a specific geographic area generally disjoint from the area covered by any other district, electors have two votes. They can cast these two votes in any manner they wish, short of giving both votes to the same candidate. Thus, in a three-candidate race, between candidates 1, 2, and 3, there are six ballots that a voter can legally cast: those voting for 1 and 2, for 1 and 3, or for 2 and 3—these being all the "double ballots"; and those voting for 1 alone, 2 alone, or 3 alone—these being all the "single ballots," or, to use the British term, "plumpers." We shall designate a ballot by enclosing in curly brackets the candidates voted for; thus, a ballot for 1 and 2 is $\{1, 2\}$, one for 3 alone is $\{3\}$, and so on. The outcome of the election depends only on the total number of votes each candidate receives. If we designate by N_{12} the number of $\{1, 2\}$ ballots cast at our hypothetical election, and adopt similar conventions for the other ballots, the candidate totals can be expressed as follows:

$$T_1 = N_1 + N_{12} + N_{13}$$
$$T_2 = N_2 + N_{12} + N_{23}$$
$$T_3 = N_3 + N_{13} + N_{23}$$

Those two candidates receiving the most and the second-most votes are awarded seats in the legislature.

It should be noted at this point that the double election with three candidates is almost identical to what might be called negative approval voting. Under this latter system, each voter would vote *against* as many candidates as he or she wished, with the candidate receiving the most of these negative votes being chosen to remain out of public life. In other words, instead of viewing the double election as the selection of two MPs, one can view it as the selection of one private citizen, in which case it is essentially equivalent to approval voting. For, the options facing a voter in the negative approval scheme are essentially the same as those facing the voter in the double election. Voters in the double election who vote $\{j\}$ can achieve the same effect in the approval election by voting against everyone but j. Similarly, those who vote $\{j, k\}$ can equivalently vote against the remaining candidate, while abstention in the double election is equivalent to abstention or to voting against all three candidates in the approval election. The vote totals in the approval election can be written (assuming that no one casts votes against all three candidates):

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$$T_1^* = N_2 + N_3 + N_{23}$$
$$T_2^* = N_1 + N_3 + N_{13}$$
$$T_3^* = N_1 + N_2 + N_{12}$$

from which it can be seen that

$$T_1^* - T_2^* = T_2 - T_1$$
$$T_1^* - T_3^* = T_3 - T_1$$
$$T_2^* - T_3^* = T_3 - T_2.$$

That is, the order of finish in the double election is exactly the reverse of the order of finish in the approval election, so that the outcome is indeed the same.

A Model of Electoral Behavior in the Double-Member District

Now that a basic familiarity with the conditions of voting in double-member districts has been established, the axioms of a theory of electoral choice can be stated. The first axiom is that citizens' preferences are defined solely over the outcomes of elections. To carry on the example of the last section, the three possible outcomes are simply the victory of candidates 1 and 2, the victory of candidates 1 and 3, and the victory of candidates 2 and 3—denoted (1, 2), (1, 3), and (2, 3), respectively. The axiom implies that voters do not care about the vote(s) they cast per se, but only about the effects of these votes on the outcome. An example of a violation of this axiom would be a situation in which a voter was offered money to vote in a particular way: for then, presumably, holding the outcome constant, the voter would prefer to vote as bribed and collect the money. The second axiom is a rationality assumption: that citizens act so as to maximize their expected utility.

Some consequences of these two axioms follow by considering an elector who prefers the outcome (1, 2) to (1, 3), and both to (2, 3). (Assuming that the voter's preferences can be represented by a Von Neumann–Morgenstern utility function, we have $u(1, 2) > u(1, 3) > u(2, 3)$.) This voter will never vote for candidate 3, since the only effect such a vote can have is to change the outcome from (1, 2) to either (1, 3) or (2, 3) and the voter seeks neither change. Similarly, the elector will always vote for candidate 1, since the only effect such a vote can have is to change the outcome from (2, 3) to either (1, 3) or (1, 2). Hence the voter will cast one of two ballots: {1} or {1, 2}. Note that this behavior satifies the definition given by Brams of sincerity, whereby a voter votes sincerely "if and only if, whenever he votes for some candidate, he votes for all candidates preferred to that candidate" (Brams, 1982, p. 10).[1]

[1] Preferences in this model have been defined only over outcomes, not candidates. Extending preferences to candidates in such a fashion that the definition of sincerity is satisfied is straightforward, however, if it is assumed that the utility of outcome (i, j) is simply the sum of the utility of having candidate i elected plus the utility of having candidate j elected:

$$u(i, j) = v(i) + v(j) \qquad \text{for all } i, j$$

where v: $\{1, 2, 3\} \rightarrow R$ is the utility function defined over candidates. With this assumption it is easy to see that $v(1) > v(2) > v(3)$, and voting for 1 or for 1 and 2 satisfies the definition of sincerity.

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If the elector knows how everyone else will vote, then he has a simple problem; he needs only calculate what the outcome would be if he voted {1}, what it would be if he voted {1, 2}, and choose the ballot leading to the preferred outcome. For most configurations of how everyone else has voted, the two ballots would be equivalent. For example, if we let T_j stand for the total number of votes received by candidate j exclusive of the votes of the elector under consideration, and $M = \min\{T_2, T_3\}$, then when $M > T_1 + 1$, both 2 and 3 are "out of reach"—the outcome will be (2, 3) no matter what ballot the elector casts.

Using the notation just introduced, 12 mutually exclusive and collectively exhaustive "states of the world" can be defined. The state S_0 is defined as the situation in which $M > T_1 + 1$; the other states are defined in Table 1. The entries in this table are ordered triples of probabilities (q_1, q_2, q_3), where q_1 is the probability that (1, 2) will be the outcome, q_2 is the probability that (1, 3) will be the outcome, and q_3 is the probability that (2, 3) will be the outcome. For example, the definition of state S_1 is $M = T_1 + 1$ and $T_3 > T_2$, which is equivalent to $T_3 > T_2 = T_1 + 1$. If the elector votes {1} in this situation, he brings about a tie between 1 and 2, with 3 a clear winner. As we assume that ties are broken equiprobably, both 1 and 2 have a probability of one-half of being returned along with 3, which implies that $q_1 = 0$, $q_2 = ½$, and $q_3 = ½$.

Each of the voter's two admissible (i.e., undominated) voting strategies yields different payoffs depending on which of the 12 states of the world (i.e., configurations of how everyone else has voted) obtains. The assumption that voters maximize their expected utility means, first, that electors have beliefs about how other electors will cast their ballots which can adequately be modeled as subjective probabilities over the various states of the world and, second, that electors attempt to take their beliefs about how others will vote into account when voting in a fashion which can adequately be modeled by saying that they choose that strategy which yields the highest expected utility.

Thus, the elector's choice between {1} and {1, 2} depends on the expected utility of the various probabilistic outcomes in Table 1. For notational simplicity, assume that the utility function, u, is normalized in the standard fashion so that $u(1, 2) = 1$, $u(2, 3) = 0$, and $u(1, 3) = b$, where $0 < b < 1$. The expected payoffs to both strategies can now be computed simply by taking the scalar product of $(1, b, 0)$ with the entries in Table 1. The results are shown in Table 2.[2]

As noted above, it is assumed that the elector has beliefs about how the other electors will cast their ballots. These beliefs are represented in the model by a 12-tuple of subjective probabilities.

$$P = (P_0, P_1, P_2, P_3, P_4, P_5, P_6, P_7, P_8, P_9, P_{10}, P_{11}),$$

[2] A similar analysis of voting, dealing with a special case under approval voting, can be found in Brams and Fishburn (1983, pp. 88–89).

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TABLE 1

Outcomes Matrix for a Three-Candidate Two-Seat Election

	States of the World											
	$M > T_1 + 1$	$M = T_1 + 1$			$M = T_1$				$M \leq T_1 - 1$			
					$T_3 > T_2$				$T_3 > T_2$			
	S_0	S_1	S_2	S_3	S_4	S_5	S_6	S_7	S_8	S_9	S_{10}	S_{11}
Acts		$T_3 > T_2$	$T_3 = T_2$	$T_3 < T_2$	$T_3 > T_2 + 1$	$T_3 = T_2 + 1$	$T_3 = T_2$	$T_3 < T_2$	$T_3 > T_2 + 1$	$T_3 = T_2 + 1$	$T_3 = T_2$	$T_3 < T_2$
{1}	(0, 0, 1)	(0, ½, ½)	(⅓, ⅓, ⅓)	(½, 0, ½)	(0, 1, 0)	(0, 1, 0)	(½, ½, 0)	(1, 0, 0)	(0, 1, 0)	(0, 1, 0)	(½, ½, 0)	(1, 0, 0)
{1,2}	(0, 0, 1)	(0, 0, 1)	(½, 0, ½)	(½, 0, ½)	(0, ½, ½)	(⅓, ⅓, ⅓)	(1, 0, 0)	(1, 0, 0)	(0, 1, 0)	(½, ½, 0)	(1, 0, 0)	(1, 0, 0)

NOTE: T_1, T_2, and T_3 indicate the vote totals of the three candidates exclusive of the vote(s) of the voter under consideration. M equals the minimum of T_2 and T_3. The entries give the probability of each possible outcome in order; for example, the entry (0, ½, ½) indicates that the probability of 1 and 2 winning is zero, the probability of 1 and 3 winning is ½, and the probability of 2 and 3 winning is ½.

TABLE 2

Payoff Matrix for a Three-Candidate Two-Seat Election

	States of the World											
Acts	S_0	S_1	S_2	S_3	S_4	S_5	S_6	S_7	S_8	S_9	S_{10}	S_{11}
{1}	0	$\frac{b}{2}$	$\frac{b+1}{3}$	$\frac{1}{2}$	b	b	$\frac{b+1}{2}$	1	b	b	$\frac{b+1}{2}$	1
{1,2}	0	0	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{b}{2}$	$\frac{b+1}{3}$	1	1	b	$\frac{b+1}{2}$	1	1

NOTE: Entries are the normalized utility of the outcome occurring when the voter takes the action specified at the left and the state of the world is as specified in the columns at the top. For the definition of the states of the world, see Table 1.

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$$P_j \geq 0, \quad \sum_{j=0}^{11} P_j = 1$$

where P_0 is the elector's subjective probability that S_0 will hold and the other P_j are defined analogously. The assumption of expected utility maximization postulates that the citizen will choose that act with the highest associated expected utility.

In the present instance, the elector will choose {1} over {1, 2} if the expected utility of {1} exceeds the expected utility of {1, 2}; that is, if

$$p_1\left(\frac{b}{2} - 0\right) + p_2\left(\frac{b+1}{3} - \frac{1}{2}\right) + p_4\left(b - \frac{b}{2}\right)$$
$$+ p_5\left(b - \frac{b+1}{3}\right) + p_6\left(\frac{b+1}{2} - 1\right) + p_9\left(b - \frac{b+1}{2}\right)$$
$$+ p_{10}\left(\frac{b+1}{2} - 1\right) > 0$$

This condition simplifies to[3]

$$b > \frac{\frac{2r(2)}{3} + r(1)}{r(1) + \frac{4r(2)}{3} + r(3)} \tag{1}$$

where $r(1) = P_6 + P_9 + P_{10}$ = the probability that candidate 1 will win regardless of the voter's choice and the only effect of the vote for 2 will be to decrease the chances that 3 is elected; $r(3) = P_1 + P_4$ = the probability that candidate 3 will win regardless of the voter's choice and the only effect of the vote for 2 will be to decrease the chances that 1 is elected; and $r(2) = (\frac{1}{2})P_2 + P_5$, which reflects the probability that the voter's choice will make or break a three-way tie and the vote for 2 will decrease both the probability that 1 and the probability that 3 is elected.

Inequality (1) is somewhat complicated, but nonetheless yields a number of intuitive and previously unnoticed implications. For expositional purposes, it may be easiest first to ignore the $r(2)$ term. This term is roughly the probability that

[3] We have, simplifying the inequality in the text,

$$p_1\left(\frac{b}{2}\right) + p_2\left(\frac{2b-1}{6}\right) + p_4\left(\frac{b}{2}\right) + p_5\left(\frac{2b-1}{3}\right)$$
$$+ p_6\left(\frac{b-1}{2}\right) + p_9\left(\frac{b-1}{2}\right) + p_{10}\left(\frac{b-1}{2}\right) > 0$$

or

$$b\,\frac{r(3)}{2} + r(2)\left(\frac{2b-1}{3}\right) + r(1)\left(\frac{b-1}{2}\right) > 0$$

which can be manipulated to yield inequality (1).

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the voter's choice will make or break a three-way tie and is presumably of second-order significance compared even to probabilities of two-way ties. If $r(2)$ is set to zero, the condition for plumping can be rewritten

$$r(1)(1 - b) - r(3)b < 0 \tag{2}$$

This exposes the basic logic of decision quite clearly. The left-hand side of the inequality can be interpreted as the net utility change caused by voting for candidate 2. The first term, $r(1)(1 - b)$, is (roughly) the probability that the vote for 2 will defeat candidate 3 times the utility *benefit* that this would bring about (changing the outcome from (1, 3) to (1, 2) yields a benefit $u(1, 2) - u(1, 3) = 1 - b$). The second term, $-r(3)b$, is (roughly) the probability that the vote for 2 will defeat candidate 1 times the utility *loss* this would incur (changing the outcome from (1, 3) to (2, 3) yields a loss of $u(2, 3) - u(1, 3) = 0 - b$). If the net utility impact of voting for 2 is negative, as in inequality (2), then the voter will plump for candidate 1. If the inequality is reversed, then he will vote for 1 and 2. Clearly, the larger $r(1)$ is relative to $r(3)$, i.e., the more likely it is that the elector's vote for 2 will be useful, helping to bring about a desired result—the defeat of 3—rather than an undesired result—the defeat of 1, the more likely the voter is to cast a double vote for 1 and 2. Similarly, the larger $1 - b$ is relative to b, i.e., the greater the potential gain to be won by defeating 3 is relative to the potential loss to be suffered by defeat of 1, the more likely the voter is to vote for both 1 and 2.

The same basic logic holds in the more general case (1), as may be indicated by considering two extreme cases. Suppose first that the elector is certain that candidate 1 will win one of the seats regardless of how he (the elector) votes. Given this expectation, the contest is essentially reduced to a two-candidate race for the remaining seat. The only possible effect of a vote for 2, given the voter's beliefs, is to defeat 3. Hence, since defeating 3 is desirable, if there is any chance that the vote will be decisive, the voter will cast a {1, 2} ballot. (In terms of the model, the certainty of 1's victory means $r(2) = r(3) = 0$; the chance of decisiveness means $r(1) > 0$; and inequality (1) reduces to $b > 1$, which is never satisfied.) If, on the other hand, the elector is certain that 3 will win one of the seats, the only possible effect of a vote for 2 is to defeat 1. Accordingly, if there is a chance of decisiveness, the elector will plump for 1. (In terms of the model, the certainty of 3's victory means $r(2) = r(1) = 0$; the chance of decisiveness means $r(3) > 0$; and inequality (1) reduces to $b > 0$, which is always satisfied.)

These two examples of the use of condition (1) represent feasible or believable thought processes. Even if one does not believe that real-world electors fully recognize all the various states of the world identified in the model, nor that they explicitly define subjective probabilities over these states and calculate expected utilities, it seems quite plausible that they might have beliefs about who was likely to win, and that they could then draw conclusions such as just discussed as if they were using the condition (1).

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Some Testable Hypotheses

The results of the previous section show that electoral choices made in double elections are, in a sense, inherently strategic. Since there is in general no dominant strategy, the probability beliefs of electors play an integral role in their choices. Thus, for example, two electors with identical preferences but different probability beliefs may make different decisions. This is a simple point, but it is worth emphasizing since the arguments made by advocates of approval voting concerning "sincerity" may be misleading if this point is not kept in mind. For example, it has been argued that "sincere strategies are essentially strategies that directly reflect the true preferences of a voter" (Brams and Fishburn, 1978, p. 837). Clearly, this should not be interpreted as meaning that sincere strategies reflect *only* preferences. Similarly, Brams and Fishburn note that "if a candidate voted for by a sincere voter should win, the sincere voter can rest assured that s/he could not have brought about the election of a preferred candidate by choosing a different admissible strategy" (Brams and Fishburn, 1978, pp. 837–38). Taken out of context, this statement is simply false: the possibility always exists, for example, that a sincere voter voting for 1 and 2 may end up wishing that he had voted only for 1 (e.g., in states S_1 or S_4).[4]

These interpretational caveats regarding sincerity are relevant to approval elections as well as double elections. This conclusion follows either from an acceptance of the equivalence of double and negative approval voting, or from a direct consideration of approval voting. Consider, for example, two voters with identical preferences in an approval election with three candidates. If one of these voters believes that his lowest-ranked candidate is nearly certain to lose, and that it is a close race between the other two candidates, he should obviously approve only his top preference. If the other voter believes that the top-ranked candidate is nearly certain to lose, he should vote for both his top and second preference. If, in the event, the second- and top-ranked candidates tie, the voter certainly cannot "rest assured that s/he would not have brought about the election of a preferred candidate by choosing a different admissible strategy." The only situation in which probability beliefs do not play a role in electoral choice is when preferences are "dichotomous"—that is, when there are two subsets of candidates such that the voter prefers any member of the "top" subset to any member of the "bottom" subset, and is indifferent between any two members of the same subset. As shown by Brams and Fishburn, when preferences are dichotomous, approval voting is strategy-proof (Brams and Fishburn, 1978, p. 845).

While strategy—that is, the use of probability beliefs in addition to preferences in arriving at electoral decisions—is theoretically endemic to both double-member and approval elections when preferences are not dichotomous, the possibility still remains that strategic behavior is unimportant in practice. After all,

[4] Brams has suggested that the quoted statement be interpreted in context as ". . . the election of a preferred candidate *not voted for* by choosing . . ." (personal communication).

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preferences might in fact be dichotomous. As regards double-member elections in the United States, for example, most of these involve four candidates—two Republicans and two Democrats—and the assumption seems reasonable that preferences are determined by party cues and hence are dichotomous for many voters. Consequently, these elections should be relatively strategy-free. In other situations, however, strategic behavior seems more likely. If approval voting were adopted for presidential primaries as advocated, for example, voters would no longer have a simple party cue to dichotomize preferences. Similarly, in three-candidate double-member elections where voters "vote for the man," strategic behavior may be important. In the remainder of the paper, attention will focus on this latter kind of election: three-candidate elections held in England under the provisions of the first Reform Act (1832). As discussed in another work (Cox, 1983), early Victorian electors were not yet as firmly attached to the two great parties as they were to become, instead voting in a more or less candidate-oriented fashion. There are other advantages to studying nineteenth century England. Not only were there a large number of English double-member districts before 1867 (193, or 76 percent of all constituencies), but elections in these districts were sometimes documented in a particularly detailed fashion: by what we refer to as a *ballot count*. A ballot count documented an election by reporting the number of each kind of ballot cast. For example, in a contest between A, B, and C, a ballot count would report the number of ballots for A and B, for A and C, for B and C, and for A alone, B alone, and C alone. Using ballot counts, one can test hypotheses focusing on the probability beliefs of the electorate.

Before stating these hypotheses, a brief discussion of the applicability of the model to the nineteenth century British case is in order. Although the second axiom is defensible—indeed, one of the purposes of this exercise is to show the usefulness of expected utility models for other nations and times—the first axiom poses some interpretational problems. The central feature of British electoral politics in the period between the first (1832) and third (1884) Reform Acts was political "influence": the transference of socioeconomic authority or creditorship into control of votes (Gash, 1977, chap. 8). In addition to the exertion of influence, the outright purchase of votes was also at times important. Neither of these phenomena, however, jibe with the first assumption. Neither a tenant-at-will voting as his landlord instructed (a classic example of electoral influence), nor an elector voting as he was paid to, has "preferences defined solely over the outcomes" of the election: both are responding to vote-contingent utilities. Although this does mean that the model is not applicable to influenced or bribed voters, this is probably not a serious flaw. This is because the nature of influence and bribery seems to have been "complete"; that is, to take the example of bribery, *both* votes of the voter were purchased (Nossiter, 1975, p. 179). What this means is that influence and bribery simply magnified the decisions of those who had influence or money. If bribery and influence were complete, in other words, then the full electorate can be reduced to a smaller "independent" electorate which satisfies the axioms. Since it is only members of this independent electorate who

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make effective decisions, a model which predicts their decisions can also predict the decisions of the larger electorate.

Assuming the model's applicability, the remaining sections focus on a data set documenting 460 three-candidate elections held between the first (1832) and second (1867) Reform Acts.[5] This data set includes virtually all three-candidate contests held at general elections in the 1832–67 period for which ballot counts exist. The only exceptions are those rare contests involving either three candidates of the same party or three candidates each of a different party, both of which pose technical difficulties for the test to be proposed. The contests which are included are dubbed "two-against-one," since they pitted two candidates of one party (referred to as the "majority" party) against a lone member of another party (referred to as the "minority" party). Such two-against-one contests were by far the most common type of contest in the period between the first and second Reform Acts. Generally, around 50 percent of the double-member districts were contested in this fashion, with the remainder more or less evenly divided between four-candidate contests and no contests at all.

A particular example of a two-against-one contest would pit two Conservatives, C_1 and C_2, against a single Liberal, L. The possible outcomes for such an election can be designated (C_1, C_2), (C_1, L) and (C_2, L). By examining each of the possible preference orderings over these three outcomes, and the responses of a typical voter holding each ranking to a change in the probability that the minority candidate will win a seat, testable consequences of the theory can be derived.

Consider first those electors who might be called *Conservative partisans* and who rank the outcomes as either $(C_1, C_2) > (C_1, L) > (C_2, L)$ or $(C_1, C_2) > (C_2, L) > (C_1, L)$ (where the symbol ">" is used to denote strict preference). If the candidate's labels in these cases are mapped into the notation developed earlier, it is seen that for both rankings the Liberal plays the role of candidate 3. These voters will consider only ballots excluding the Liberal, viz., $\{C_1\}$, $\{C_2\}$, and $\{C_1, C_2\}$. And, as the probability P that the Liberal will win a seat increases, more Conservative partisans should make a "forced choice" and plump for one of the Conservatives: one should observe more $\{C_1\}$ and $\{C_2\}$ ballots and fewer $\{C_1, C_2\}$ ballots. On the other hand, for those voters (the *Liberal partisans*) who rank the outcomes either $(C_1, L) > (C_2, L) > (C_1, C_2)$ or $(C_2, L) > (C_1, L) > (C_1, C_2)$, the minority candidate plays the role of candidate 1. As P increases, Liberal partisans (who consider only Liberal plumpers or split votes) increasingly have the luxury of safely voting for the lesser Conservative evil, and one expects more $\{C_1, L\}$ and $\{C_2, L\}$ ballots and fewer $\{L\}$ ballots.

[5] Three sources were used to collect these ballot counts: (1) Henry Stooks Smith, *The Parliaments of England* (London: Simpkin, Marshall and Co., 1844–50), available in a reprinted edition edited by F. W. S. Craig (Chichester: Political Reference Publications, 1973); (2) W. W. Bean, *The Parliamentary Representation of the Six Northern Counties of England* (Hull: Printed for the Author by Charles Henry Barnwell, 1890); (3) the collections of poll books in the Institute of Historical Research (University of London), the Guildhall Library (City of London), and the British Library.

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For all other preference orderings, the result of an increase in P is either ambiguous or nil. For voters (the *Independents,* for lack of a better term) with the strict rankings $(L, C_1) > (C_1, C_2) > (L, C_2)$ and $(L, C_2) > (C_1, C_2) > (L, C_1)$, the Liberal candidate plays the role of candidate 2. An increase in P for these voters tends to reduce the choice to one between the two Conservatives, but this choice is already clear since one corresponds to candidate 1 and the other to candidate 3. Hence, there is no clear prediction to make; whether such voters plump or cast double ballots depends on other factors. All voters who are indifferent between any two of the outcomes have dominant strategies and thus do not respond to the probability that the minority candidate will win a seat. Consider, for example, an elector indifferent between (C_1, L) and (C_2, L), and preferring both to (C_1, C_2). With these dichotomous preferences, the choice is essentially binary, and the dominant strategy is to vote for L alone. Finally, voters totally indifferent will obviously not respond to changes in P.

In sum, then, there are two conditional predictions about the electoral effect of an increase in the probability that the minority candidate in a two-against-one contest will win. First, if certain kinds of preferences (similar to those associated with Conservative partisans in the example) are prevalent, then increases in P should lead to an increase in *majority plumping:* the casting of single votes for one of the majority party's two candidates, and a decrease in *party voting:* double votes for the majority candidates. Second, if certain other kinds of preferences (similar to those linked with Liberal partisans in the example) are prevalent, then increases in P should lead to an increase in *split voting:* the casting of votes for both parties, and a decrease in *minority plumping:* single votes for the minority candidate. If neither of these kinds of preferences is common, then no effect is expected from an increase in P. Since preferences other than those associated with Liberal or Conservative partisans simply add "noise" to the predictions, it may be necessary to test the predictions in terms of ratios. Somewhat weaker predictions are that increases in P should lead to (1) an increase in the ratio of majority plumping to party voting and (2) an increase in the ratio of split voting to minority plumping.

Results

What is needed in order to test the predictions of the last section is some way to measure the general perception in a given constituency of the probability that the minority candidate will win a seat. One way to do this is simply to look at the actual outcome. If voters' beliefs about what the outcome *would be* bore a relationship to what it actually *turned out to be,* then we ought to expect that voters in districts where the minority candidate topped the poll ("type 1 outcome") generally estimated high probabilities of this candidate winning a seat; that voters in districts where the minority candidate placed third ("type 3 outcome") generally felt beforehand that his chances were slim; and that voters in districts where the minority candidate came in second ("type 2 outcome") generally had intermediate beliefs about the probability of his securing a seat. To the

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extent that these classifications capture the relevant probability, we expect the highest rate of nonpartisan voting (i.e., split voting and majority plumping) in the type 1 districts, an intermediate rate in the type 2 districts, and the lowest rate in the type 3 districts, with just the opposite expectations regarding partisan voting (i.e., party voting and minority plumping).

Unfortunately, the outcome type of a contest also reflects the number of voters holding each kind of preference ranking. That the minority candidate finished first rather than last presumably indicates (to carry on the two Conservative versus one Liberal example) that there were relatively more Liberal partisans and fewer Conservative partisans. It is therefore to be expected that more ballots of the kinds Liberal partisans consider (viz., $\{L\}$, $\{C_1, L\}$, and $\{C_2, L\}$ ballots) will be cast and that, similarly, fewer "Conservative" ballots (viz., $\{C_1\}$, $\{C_2\}$, and $\{C_1, C_2\}$) will be cast. Hence, the theoretical expectations that there be more split voting ($\{C_1, L\}$ and $\{C_2, L\}$) and less party voting ($\{C_1, C_2\}$) are to be expected in any event. But the predictions that there will be more majority plumping ($\{C_1\}$ and $\{C_2\}$) and less minority plumping ($\{L\}$) run *counter* to the artifactual expectations just noted. And there are *no* artifactual expectations concerning the two ratios mentioned at the end of the previous section. Consequently, the strongest evidence for or against the theory will come from these latter four statistics.

The actual patterns observed in English elections during the period of the first Reform Act are presented in Table 3. As expected, both theoretically and artifactually, the split voting rate was highest in type 1 districts: 23.4 percent of all voters in type 1 contests cast split votes, a figure more than twice the corresponding rate in type 3 districts (the rate in type 2 districts being intermediate). Also as expected, there was a steady decline in party voting from type 3 through type 2 to type 1 contests. More crucial for the theory, majority plumping rates also conform strongly to theoretical expectations. Whereas only 4.7 percent of all

TABLE 3

Voting in English Double-Member Elections, 1832–65

Outcome Type	SV	MAJ	PV	MIN	NC	*N*
1	23.4%	13.5%	28.3%	34.8%	90	158,592
2	19.2	7.5	39.5	33.8	133	257,634
3	11.6	4.7	51.1	32.6	237	497,483

SOURCE: Author's calculations from data described in footnote.

NOTES: The outcome type is *j* if the minority candidate finished in the *j*th position. The column headed NC gives the number of election contests of each type, while the last column gives the number of voters involved. Thus, 90 contests of outcome type 1 occurred at general elections between 1832 and 1865 inclusive, and 158,592 voters participated in these contests. The first four columns give the percentage of the voters in each outcome type who cast split votes (SV), majority plumpers (MAJ), party votes (PV), and minority plumpers (MIN). Thus, of the 497,483 voters observed in type 3 contests, 11.6% split their votes, 4.7% cast majority plumpers, and so on. All differences in the table are statistically significant at the .01 level.

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voters in type 3 contests cast majority plumpers, the comparable figure in type 1 contests was nearly three times as high, with the rate in type 2 elections being intermediate. Finally, as to minority plumping, the theory fails to predict the observed pattern, which corresponds instead to artifactual expectations. Nonetheless, it should be noted that the minority plumping figures are nearly constant. What changes between type 1 and 3 outcomes is not so much an increase in the number of minority plumpers but rather (1) a doubling of the split voting rate, (2) a halving of the party voting rate, and (3) a tripling of the majority plumping rate, all of which is consistent with the theory. Further, if one computes the ratio of majority plumping to party voting and the ratio of split voting to minority plumping, statistically significant increases are observed in both ratios, as can be seen in Table 4.

Conclusion

The chief aim of this paper has been to elucidate the theory of electoral choice in multi-member districts. Taking the simplest case of a three-candidate election in a double-member district, the paper develops a model of electoral choice as a standard decision under risk. It is found that this simplest case is essentially equivalent to approval voting with three candidates; thus, voting in double elections with three candidates is "sincere" in the same sense that approval voting is, and a unique sincere strategy exists when preferences are dichotomous. Nonetheless, it is shown that electoral choice in double (and approval) elections is inherently strategic in the sense that voters' beliefs about how others will vote affect their decisions (when preferences are not dichotomous). This point is important because it has been claimed that sincerity of choice is an important property, guaranteeing that choices reflect preferences "directly." This paper shows that there are fairly severe restrictions on the sense in which this is true, due to the strategic nature of choice. In the later sections of the paper, an examination of actual voting behavior in double-member districts was made to see if theoretically predicted strategic behavior was evident in practice. While the particular tests made, relying as they do on aggregate data, fall far short of the sophistication that can be achieved in the study of strategic behavior by the use of surveys (cf. Cain, 1978; Black, 1978), they are the best currently available and are consistent with the existence of strategic behavior in double-member elections. Thus, not only does the theoretical potential for strategic voting clearly

TABLE 4

Conditional Voting Odds

Outcome Type	MAJ/PV	SV/MIN
1	.477	.672
2	.190	.568
3	.092	.356

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exist even when electoral choice is sincere, but it would seem that that potential is sometimes realized.

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